CC-9: Elements of modern Physics

Syllabus:

Two slit interference experiment with photons, atoms and particles; linear superposition principle as a consequence; Matter waves and wave amplitude; Schrodinger equation for non-relativistic particles; Momentum and Energy operators; stationary states; physical interpretation of a wave function, probabilities and normalization; Probability and probability current densities in one dimension. (10 Lectures)

Two slit interference Exprt:

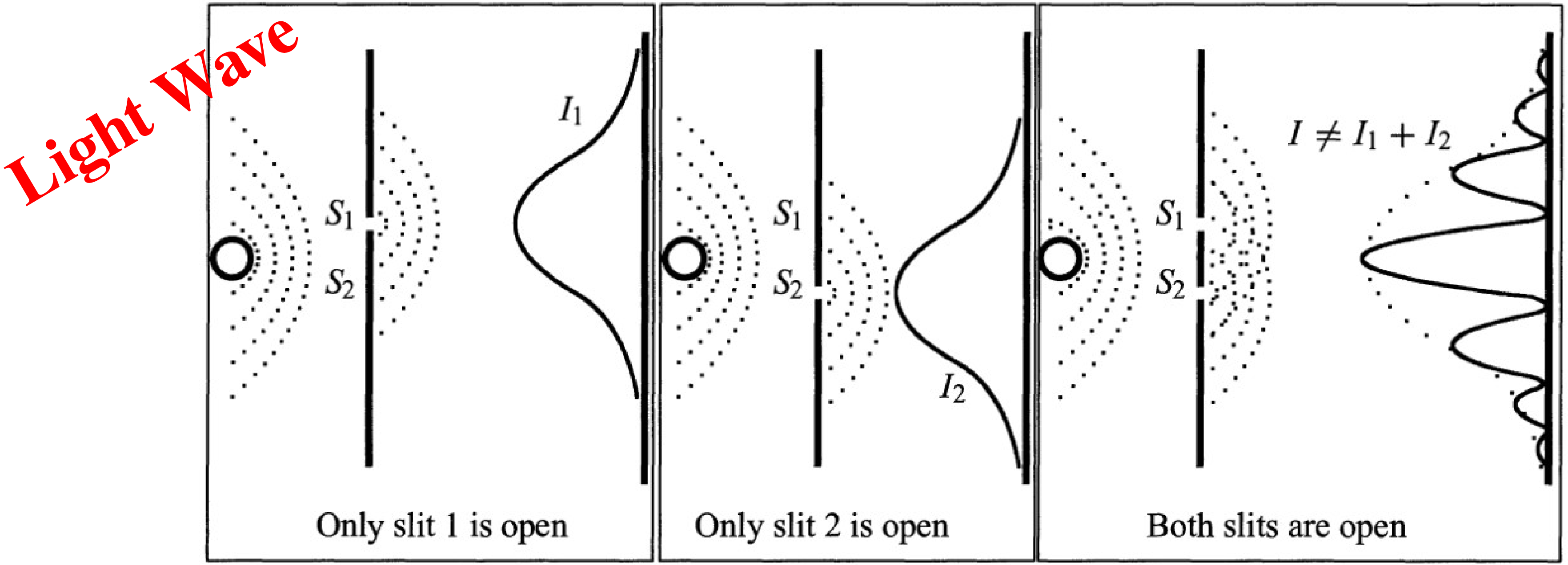

Figure 1.8 The double-slit experiment: S is a source of *waves*, I_1 and I_2 are the intensities recorded on the screen when only S_1 is open, and then when only S_2 is open, respectively. When both slits are open, the total intensity is no longer equal to the sum of I_1 and I_2; an *oscillating* term has to be added.

Figure from Zettili: QM

Two slit interference Exprt:

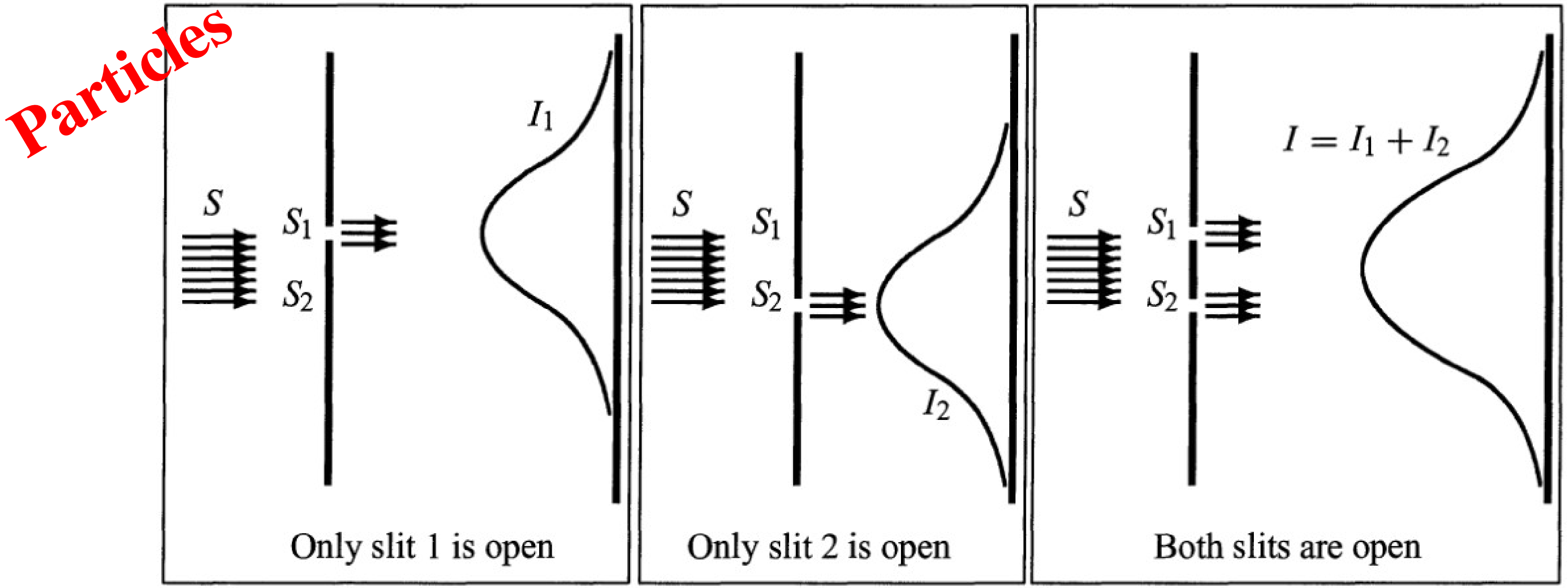

Figure 1.7 The double-slit experiment with *particles*: S is a source of *bullets*, I_1 and I_2 are the intensities recorded on the screen, respectively, when only S_1 is open and then when only S_2 is open. When both slits are open, the total intensity is $I = I_1 + I_2$.

Figure from Zettili: QM

Two slit interference Exprt:

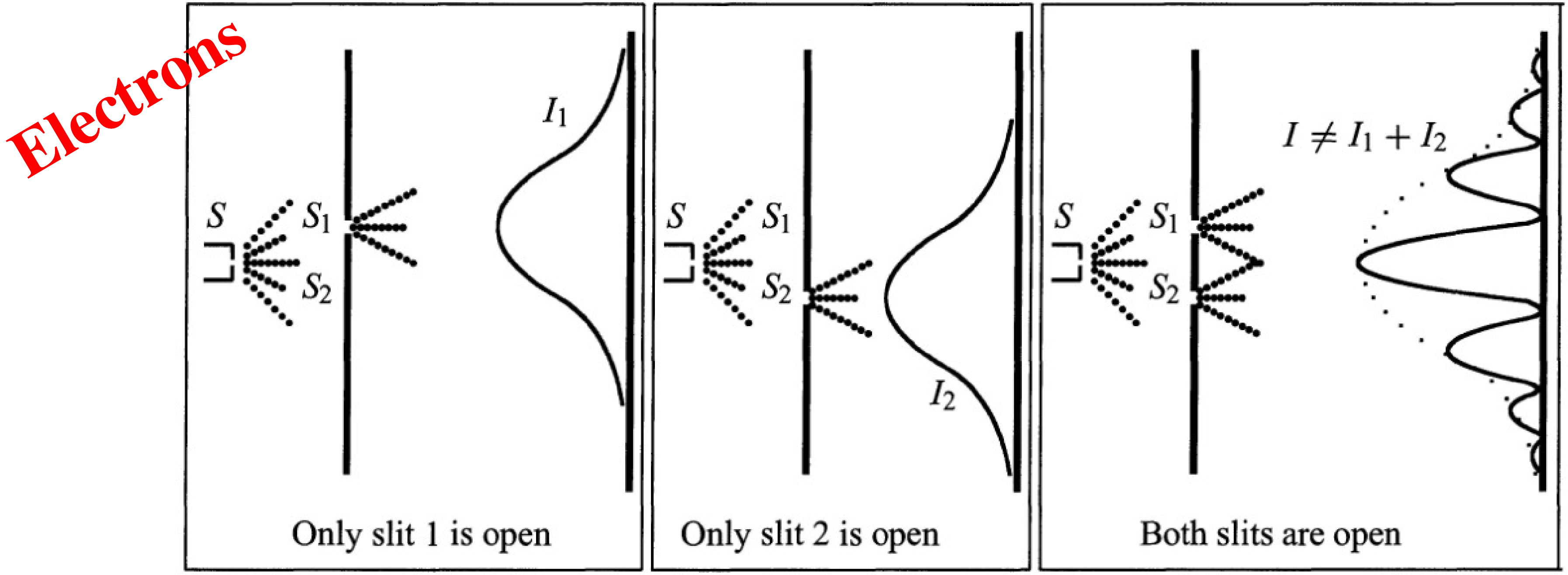

Figure 1.9 The double-slit experiment: S is a source of *electrons*, I_1 and I_2 are the intensities recorded on the screen when only S_1 is open, and then when only S_2 is open, respectively. When both slits are open, the total intensity is equal to the sum of I_1, I_2 and an *oscillating* term.

Figure from Zettili: QM

Two slit interference Exprt:

If we want to video record each electron then the pattern is killed! Quantum effect?

Electrons

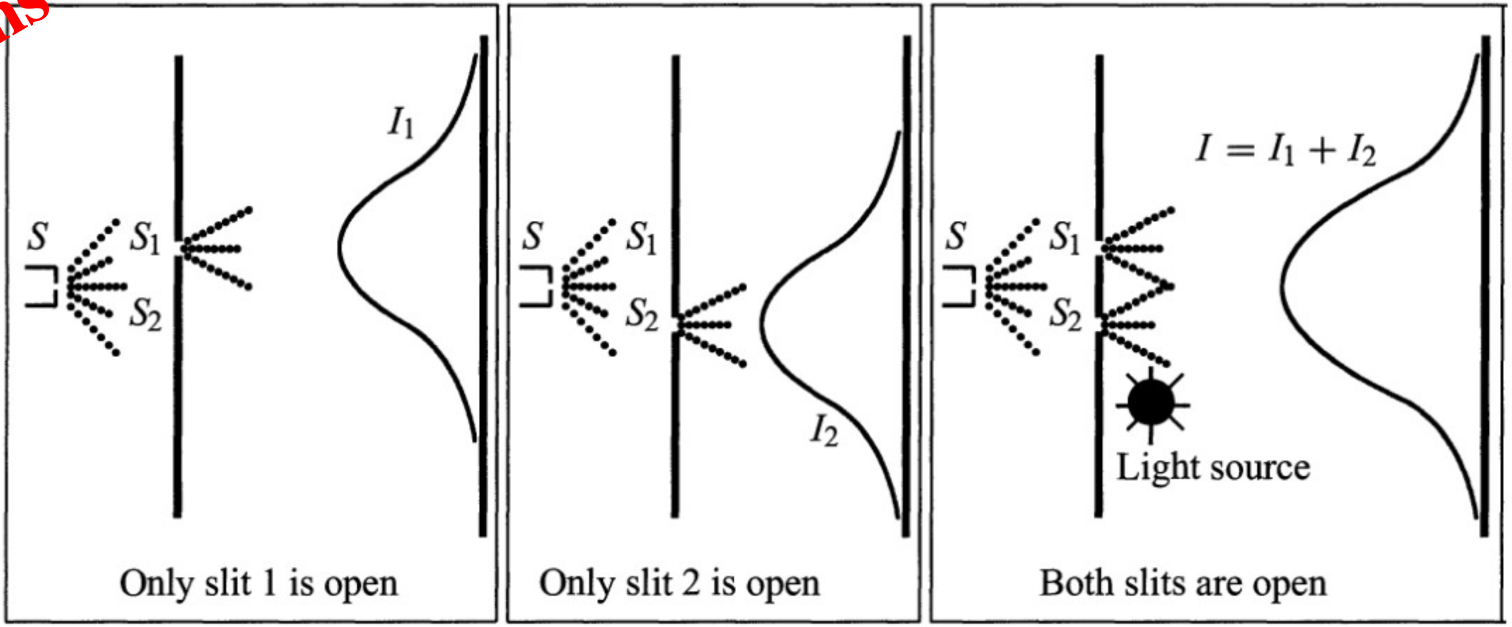

Figure 1.10 The double-slit experiment: S is a source of *electrons*. *A light source* is placed behind the wall containing S_1 and S_2. When both slits are open, the interference pattern is destroyed and the total intensity is $I = I_1 + I_2$.

Figure from Zettili: QM

Linear Superposition Principle

Principle of Linear Superposition

How do we account mathematically for the existence of the interference pattern in the double-slit experiment with material particles such as electrons? An answer is offered by the *superposition principle*. The interference results from the superposition of the waves emitted by slits 1 and 2. If the functions $\psi_1(\vec{r}, t)$ and $\psi_2(\vec{r}, t)$, which denote the waves reaching the screen emitted respectively by slits 1 and 2, represent two physically possible states of the system, then any linear superposition

$$\psi(\vec{r}, t) = \alpha_1 \psi_1(\vec{r}, t) + \alpha_2 \psi_2(\vec{r}, t)$$

also represents a physically possible outcome of the system; α_1 and α_2 are complex constants.

Linear Superposition Principle

We can summarize the double-slit results in three principles:

- Intensities add for classical particles: $I = I_1 + I_2$.
- Amplitudes, not intensities, add for quantum particles: $\psi(\vec{r}, t) = \psi_1(\vec{r}, t) + \psi_2(\vec{r}, t)$; this gives rise to interference.
- Whenever one attempts to determine experimentally the outcome of individual events for microscopic material particles (such as trying to specify the slit through which an electron has gone), the interference pattern gets destroyed. In this case the intensities add in much the same way as for classical particles: $I = I_1 + I_2$.

Matter Wave & Wave Amplitude

- The electron has a matter wave which is oscillating with de-Broglie wavelength $\lambda = h/mv$
- The properties of the electron (rather quantum particle) is described by the wave function or state function (not a function of wavelength!) ψ (x,t)
- ψ (x,t) is the Wave function whose square is the probability of finding an electron at a given instant of time at a given place.

Schrodinger Equation (Non-Relativistic)

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi.$$

Ask you teacher, to explain all terms. We did it in the class..

$$\hbar = \frac{h}{2\pi} = 1.054573 \times 10^{-34} \text{J s}.$$

m=mass of particle
V = Potential energy

Momentum & Energy Operator

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{E} = i\hbar\frac{\partial}{\partial t}$$

Momentum Operator
(how it came?)

$$\langle x \rangle = \int_{-\infty}^{+\infty} x |\Psi(x,t)|^2 \, dx.$$

$$\frac{d\langle x \rangle}{dt} = -\frac{i\hbar}{m} \int \Psi^* \frac{\partial \Psi}{\partial x} \, dx.$$

$$\frac{d\langle x \rangle}{dt} = \int x \frac{\partial}{\partial t} |\Psi|^2 \, dx = \frac{i\hbar}{2m} \int x \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right) dx.$$

$$\langle v \rangle = \frac{d\langle x \rangle}{dt}.$$

This expression can be simplified using integration by parts[10]:

$$\frac{d\langle x \rangle}{dt} = -\frac{i\hbar}{2m} \int \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right) dx.$$

$$\langle x \rangle = \int \Psi^* (x) \Psi \, dx,$$

$$\langle p \rangle = \int \Psi^* \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right) \Psi \, dx.$$

Stationary States

- A wavefunction which does not evolve with time is called Stationary states.
- When enrgy is bounded, then we get bound state.

They are **stationary states**. Although the wave function itself,

$$\Psi(x,t) = \psi(x)\, f(t),$$

$$\boxed{\Psi(x,t) = \psi(x)e^{-iEt/\hbar},}$$

They are states of *definite total energy*. In classical mechanics, the total energy (kinetic plus potential) is called the **Hamiltonian**:

$$H(x,p) = \frac{p^2}{2m} + V(x).$$

Stationary States (more info:)

- A wavefunction which does not evolve with time is called Stationary states.
- When enrgy is bounded, then we get bound state.

$$\Psi_1(x,t) = \psi_1(x)e^{-iE_1t/\hbar}, \quad \Psi_2(x,t) = \psi_2(x)e^{-iE_2t/\hbar}, \ldots.$$

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_nt/\hbar}.$$

Wave function

One RING to rule them all

Ring = Wave function

From: the movie: Lord of Rings

Probabilities & Normalization

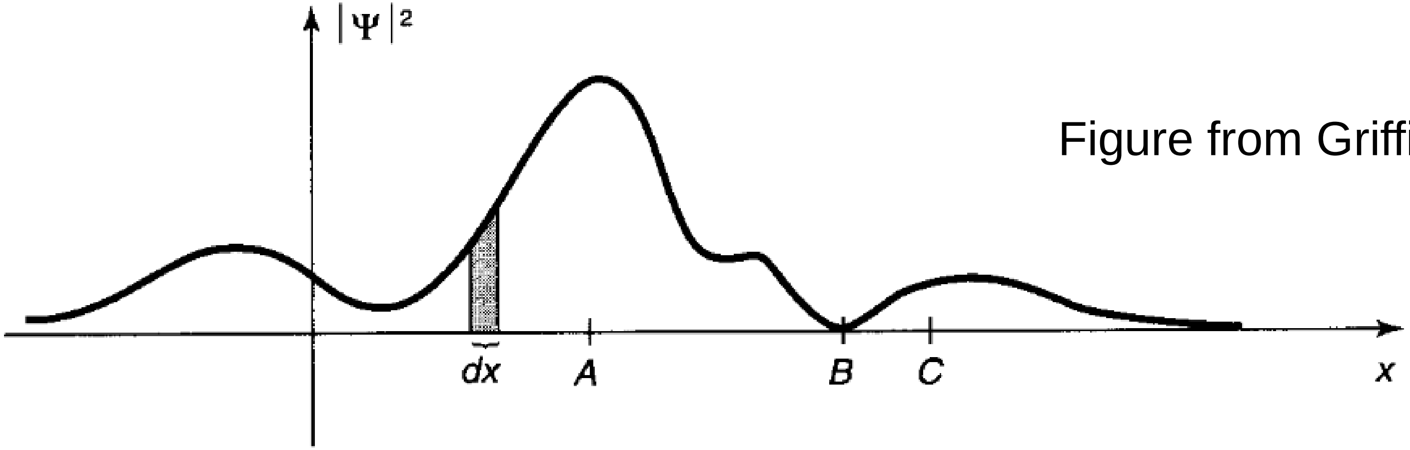

Figure from Griffiths: QM

Figure 1.2: A typical wave function. The particle would be relatively likely to be found near A, and unlikely to be found near B. The shaded area represents the probability of finding the particle in the range dx.

$$|\Psi(x,t)|^2\,dx = \left\{\begin{array}{l}\text{probability of finding the particle}\\ \text{between } x \text{ and } (x+dx), \text{ at time } t.\end{array}\right\}$$

Probability Current Density (1D)

Probability Density

Probability Current Density

$$\rho(\vec{r},t) = \Psi^*(\vec{r},t)\Psi(\vec{r},t), \qquad \vec{J}(\vec{r},t) = \frac{i\hbar}{2m}\left(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi\right);$$

Prove the continuity equation starting from Schrodinger equation in one dimension:

(Home Work problem)

$$\frac{\partial\rho(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0,$$

This is 3D result.

What is 1D form?

Read Books
Solve problems