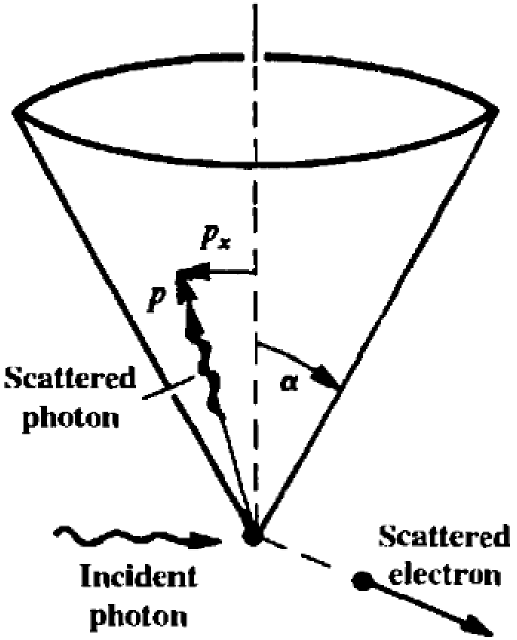
Elements of Modern Physics

- **Unit 2**

Position measurement- gamma ray microscope thought experiment; Wave-particle duality, Heisenberg uncertainty principle (Uncertainty relations involving Canonical pair of variables): Derivation from Wave Packets impossibility of a particle following a trajectory; Estimating minimum energy of a confined particle using uncertainty principle; Energy-time uncertainty principle- application to virtual particles and range of an interaction. (5 Lectures)

Gamma Ray microscope Expt



Measurement Process: Position and Momentum

$$d = \frac{\lambda}{\sin\alpha} = \Delta x$$

$$p = \frac{h}{\lambda}$$

$$\Delta p_x = \frac{h}{\lambda}\sin\alpha$$

We get the uncertainty relation as a consequence..

$$\Delta x \Delta p_x = h$$

Wave particle duality

- **Wave nature or particle nature depends on our own experiments/way of looking at it.**
- **Both properties are present in Photon. Its just they reveal what we want.**

Heisenberg Uncertainty principle

- **Uncertainty principle is involved between two canonical conjugate variables A & B, for example, position-momentum; energy-time, etc.**
- **The quantities have dimension such that the unit of their product is Joule-Sec or Action.**

This example illustrates the *Heisenberg uncertainty principle*, first set forth in 1927 by W. Heisenberg. A quantum-mechanical analysis shows that for all types of experiments the uncertainties Δx and Δp_x will always be related by

$$\Delta p_x \, \Delta x \geq \frac{h}{4\pi}$$

The Heisenberg uncertainty relation can also be formulated in terms of other conjugate variables. For example, in order to measure the energy E of a body, an experiment must be performed over a certain time interval Δt. An analysis shows that the uncertainty in the energy, ΔE, is related to the time interval Δt over which the energy is measured by

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

Thus the energy of a body can be known with perfect precision ($\Delta E = 0$) only if the measurement is made over an infinite period of time ($\Delta t = \infty$).

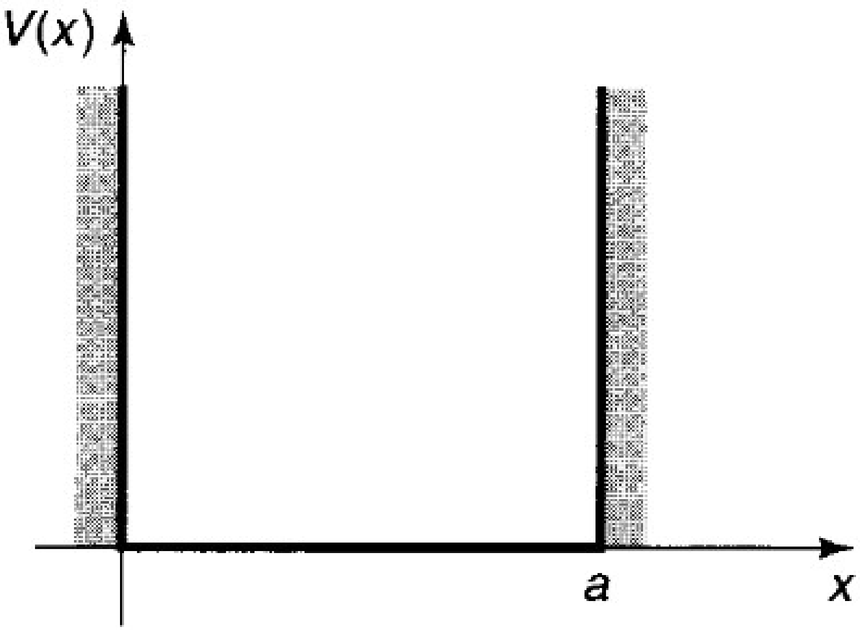
Heisenberg Uncertainty principle

- **Derivation from Wave Packets: Impossibility of a particle following a trajectory**

Important notes: Please follow the book: Modern Physics by Gatrue and Savin, solve Examples and Exercises from that book.

Heisenberg Uncertainty principle

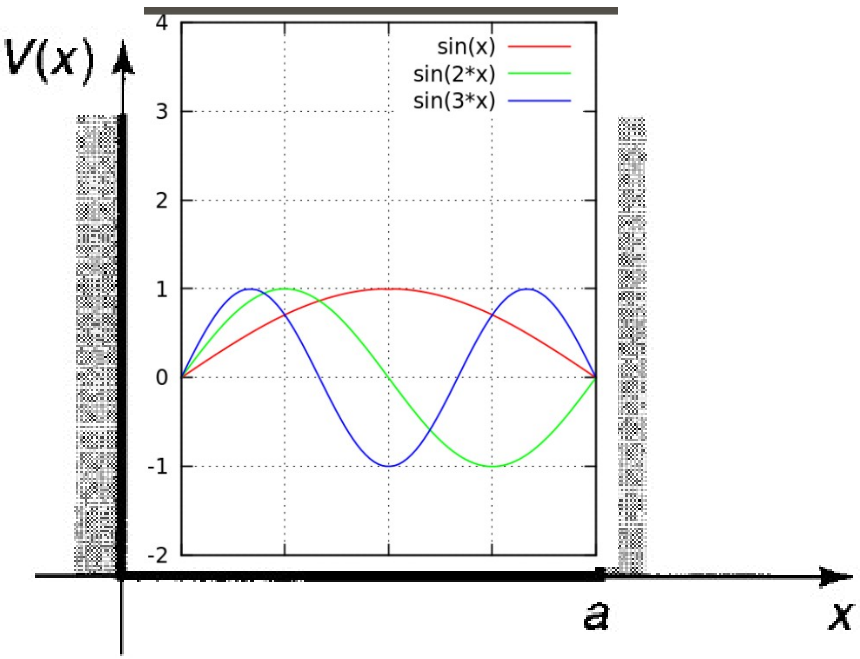
- **Estimating minimum energy of a confined particle using uncertainty principle**
- **Consider a particle is confined inside a box of length L**



If the particle is confined to a line segment, say from $x = 0$ to $x = L$, the probability of finding the particle outside this region must be zero. Therefore, the wave function ψ must be zero for $x \leq 0$ or $x \geq L$, since the square of ψ gives the probability for finding the particle at a certain location. Inside the limited region, the wavelength of ψ must be such that ψ vanishes at the boundaries $x = 0$ and $x = L$, so that it can vary continuously to the outside region. Hence only those wavelengths will be possible for which an integral number of half-wavelengths fit between $x = 0$ and $x = L$, i.e., $L = n\lambda/2$, where n is an integer, called the *quantum number*, with values $n = 1, 2, 3, \ldots$. From the de Broglie relationship $\lambda = h/p$ we then find that the particle's momentum can have only discrete values given by

Heisenberg Uncertainty principle

- **Estimating minimum energy of a confined particle using uncertainty principle**
- **Inside a box V=0**



$$p = \frac{h}{\lambda} = \frac{nh}{2L}$$

$$E = K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(nh/2L)^2}{2m}$$

$$E_n = n^2 \frac{h^2}{8mL^2} \qquad n = 1, 2, 3, \ldots$$

Energy-time uncertainty

- **Application to virtual particles and range of an interaction.**

A very nice example problem:

If an electron is confined within a nucleus whose diameter is 10^{-14} m, estimate its minimum kinetic energy.

Ans. The de Broglie wavelength of a minimum-energy electron confined inside the nucleus would be approximately twice the nuclear diameter (one-half a wavelength would fit into the diameter). Therefore, the electron's momentum would be of the order of magnitude

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{12.4 \times 10^3 \text{ eV} \cdot \text{Å}}{(2 \times 10^{-4} \text{ Å})c} = 62 \times 10^6 \frac{\text{eV}}{c} = 62 \frac{\text{MeV}}{c}$$

corresponding to a kinetic energy of

$$K = \sqrt{(pc)^2 + E_0^2} - E_0 = \sqrt{\left(62 \frac{\text{MeV}}{c} \times c\right)^2 - (0.511 \text{ MeV})^2} - 0.511 \text{ MeV} = 61 \text{ MeV}$$

Energy-time uncertainty

- **Application to virtual particles and range of an interaction.**
- **The question is what is time scale that a pion is produced in the nucleus**

$$n \rightarrow p + \pi^-$$

Consider rest mass of pion is 140 MeV

In quantum mechanics, conservation of energy can be violated in the amount $m_\pi c^2$ if the time for the process is of the order given by the Heisenberg uncertainty principle:

$$\Delta t \, \Delta E \approx \hbar \qquad \text{or} \qquad \tau_0(m_\pi c^2) \approx \hbar \qquad \text{or} \qquad \tau_0 \approx \frac{\hbar}{m_\pi c^2}$$

Therefore, strong interaction processes occur on a time scale of about 10^{-24} s.

Prove this

Thank you

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