Sem-6: CC-14: Statistical Mechanics	Quantum theory of Radiation
Instructor: Somenath Jalal	Assignment-3 (Must Do)

This questions below are taken from final year question papers of the Burdwan University.

1. (a) (i) Obtain Planck's formula for black body radiation using Bose-Einstein statistics.

   (ii) Deduce the Stefan-Boltzmann law of radiation from Planck's law.

   (b) Find out the volume of phase space of a classical linear harmonic oscillator of mass m and angular frequency ω bounded by two surfaces of constant energy E and $E + dE$.

   Part (a) - (i) and (ii) we know the answer from the notes

$$(i) \qquad u(\omega, T) = \frac{\hbar}{\pi^2 c^2} \frac{\omega^3}{e^{\frac{\hbar\omega}{k_B T}} - 1} \tag{1}$$

$$(ii) \qquad U(T) = \sigma A T^4 \tag{2}$$

   Part (b): the energy equation is

$$E = p^2/2m + 1/2m\omega^2 x^2 \tag{3}$$

   Area of phase space

$$A = \pi ab = \frac{2\pi E}{\omega} \tag{4}$$

   The new area for energy $E' = E + \Delta E$ is

$$A' = \pi ab = \frac{2\pi(E + \Delta E)}{\omega} \tag{5}$$

   Thus volume of phase space bounded by a classical linear harmonic oscillator bounded by two surfaces of constant energy E and $E + \Delta E$ is

$$\frac{2\pi \Delta E}{\omega}$$

2. Suppose there are three cells in phase space.: 1, 2 , 3. Let $N = 30$ and $N_1 = N_2 = N_3 = 10$ and $E_1 = 2$ Joules, $E_2 = 4$ Joule and $E_3 = 6$Joule. If $\delta N_3 = -2$ find δN_1 and δN_2 such that $\delta N = 0$ and $\delta U = 0$

3. State Wien's displacement law in radiation. A black body at a temperature 1646 K has the wavelength corresponding to the maximum emission (λ_m) equal to 1.78 micron. Find the temperature of the moon (assumes to be black body) if λ_m for the moon is 14 micron.

4. (a) A system with just two energy levels is in thermal equilibrium with a heat reservoir at temperature 600K. The energy gap between the levels is 0.1 eV Find

   (i) the probability that the system is in the higher energy level and

   (ii) the temperature at which the probability will equal 0.25

5. Consider a system of N non-interacting particles. Each particle may exist in either of the two energy states $E = 0$ and $E = \epsilon$.

   (a) Find the entropy S as a function of n, the number of particles in the energy state $E = \epsilon$.

   (b) Show that $S(n)$ is maximum for $n = N/2$

6. (a) Consider a system of N weakly coupled particles obeying Maxwell-Boltzmann statics, kept at a temperature T. Each particle may exist in one of the three non-degenerate levels of energy $-\epsilon,\ 0,\ \epsilon$.

   (i) What is the entropy of the system at $T = 0\ K$.

   (ii) What is the maximum possible entropy of the system?

   (iii) What is the minimum possible energy of the system?

   (iv) What is the partition function of the system?

   (v) What is the most probable energy of the system?

   (vi) if $C(T)$ be the heat capacity of the system find the value of $\int_0^a \frac{C(T)}{T} dT$

7. A cube of side 20 cm contains diatomic H_2 gas at temperature $T = 300\ K$. Mass of each H-atom in a H_2 molecule is 1.66×10^{-24} g and the distance of separation between the atoms in a molecule is 10^{-8} cm. Assume that the gas behaves like an ideal gas, ignore vibrational motion.

   (a) What is the average velocity of the molecules?

   (b) What is the average velocity of rotation of the molecules around an axis, which is the perpendicular bisector of the line joining the two atoms (consider each atom as point mass)?

   (c) Find the values of molar heat capacities C_p and C_v.

8. Choose correct answer: When the temperature of a blackbody is doubled, the maximum value of its spectral energy density, with respect to that at initial temperature, would become (a) $\frac{1}{16}$ times, (b) 8 times, (c) 16 times, (d) 32 times

9. Give the classical statistical mechanical definition of a partition function.

   Consider an ideal Boltzmann gas of N indistinguishable particles confined to a volume V at temperature T, in which the energy of each particle ϵ is related to its momentum p as $\epsilon = c\ p$, where c is a constant

   (i) Determine the partition function of the system.

   (ii) Find the pressure and the specific heat at constant volume.

   (iii) Calculate the chemical potential of the gas.