CC-3: Electricity & Magnetism

B.Sc Physics Sem – II (Honours)

Electrical Circuits: AC Circuits: Kirchhoff"s laws for AC circuits. Complex Reactance and Impedance. Series LCR Circuit: (1) Resonance, (2) Power Dissipation and (3)Quality Factor, and (4) Band Width. Parallel LCR Circuit. (4 Lectures)

Series LCR circuit

- See book Waves and Oscillation by N K Bajaj

- Resonance
- Power Dissipation
- Quality Factor
- Band Width.

First, you and I do agree that, differential equation of charge (here ***x***) for LCR circuit with an AC voltage source looks like this:

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0{}^2 x = \frac{F_0}{m} \cos \omega t$$

Inductor (L) with AC source

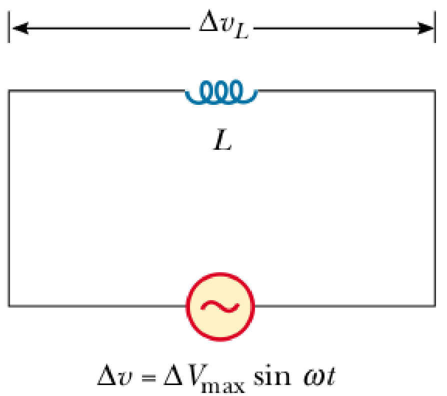
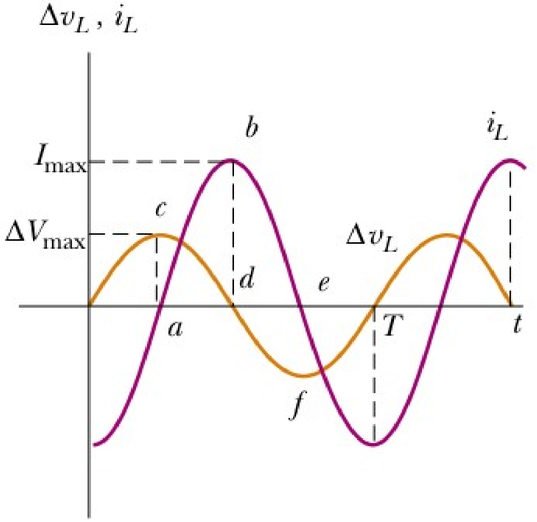


Figure 33.4 A circuit consisting of an inductor of inductance L connected to an ac generator.

$$L \frac{di}{dt} = \Delta V_{max} \sin \omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$



quantity X_L, called the **inductive reactance,** is

$$X_L = \omega L$$

Capacitor (C) with AC source

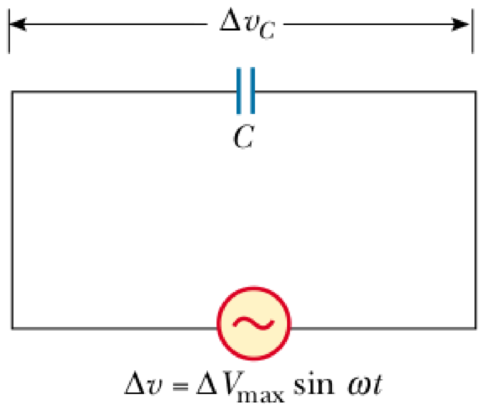


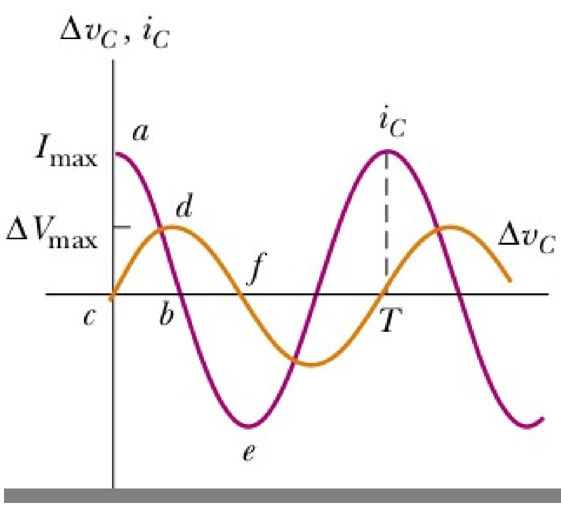
Figure 33.7 A circuit consisting of a capacitor of capacitance C connected to an ac generator.

$$\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$$

$$q = C \Delta V_{max} \sin \omega t$$

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

$$i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$



X_C is called the **capacitive reactance:**

$$X_C = \frac{1}{\omega C}$$

Series LCR circuit

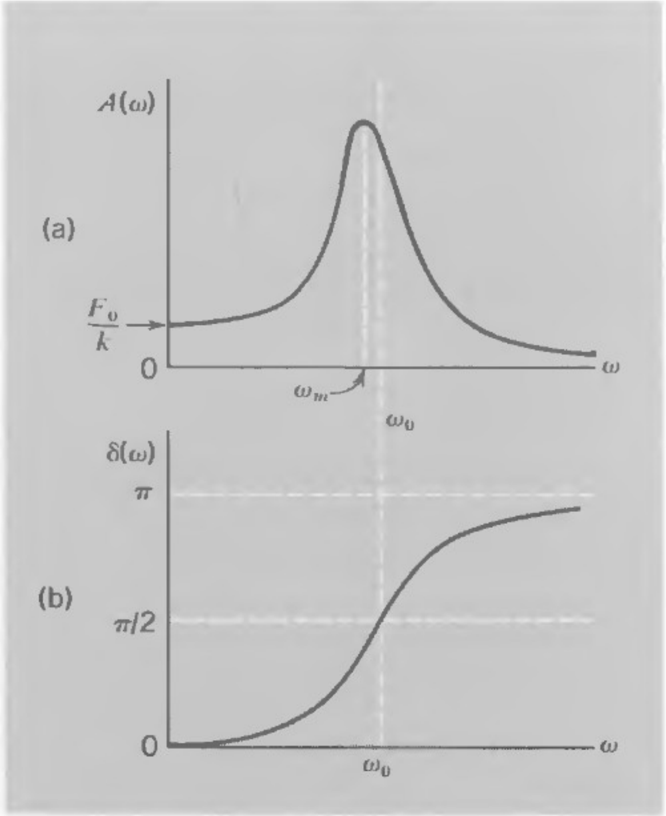
$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + {\omega_0}^2 x = \frac{F_0}{m} \cos \omega t$$

Solutions: X is charge here.. Just to remind you
Damped Harmonic Oscilaltor..

$$A(\omega) = \frac{F_0/m}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}$$

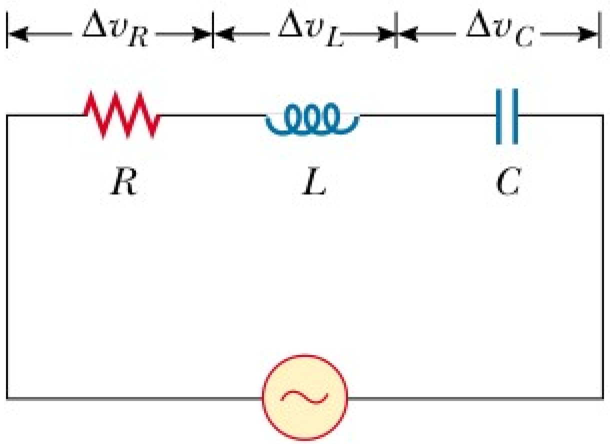
$$\tan \delta(\omega) = \frac{\gamma\omega}{\omega_0^2 - \omega^2}$$

Resonance at $\omega = \omega_0$



Please attend class, for the detailed calculation,

Series LCR circuit



The **impedance** Z of the circuit is defined as

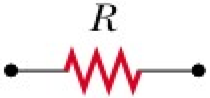
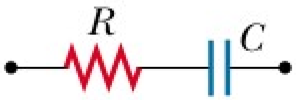
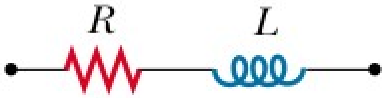
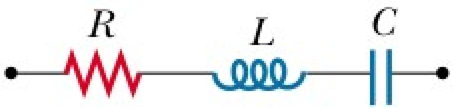
$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Plot Z as function of frequency ***w***

Plot phase angle as function of ***w***

At a glance: L-C-R

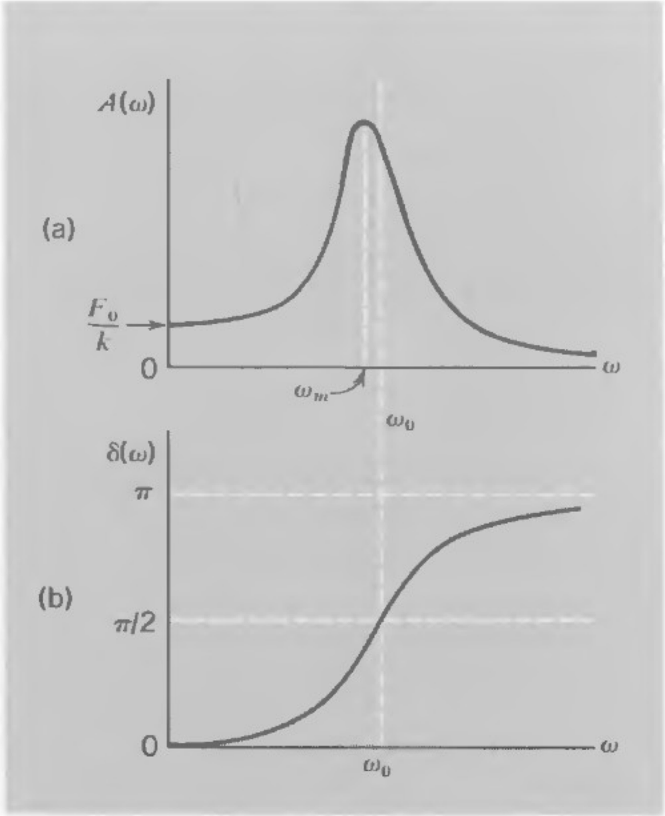
Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Series LCR circuit

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0{}^2 x = \frac{F_0}{m} \cos \omega t$$

Quality factor: $$Q = \frac{\omega_0}{\gamma}$$

$$Q = \frac{\omega_0 L}{R}$$



Please attend class, for the detailed calculation,

Series LCR circuit

- See book Waves and Oscillation by N K Bajaj

- Resonance
- Power Dissipation
- Quality Factor
- Band Width.

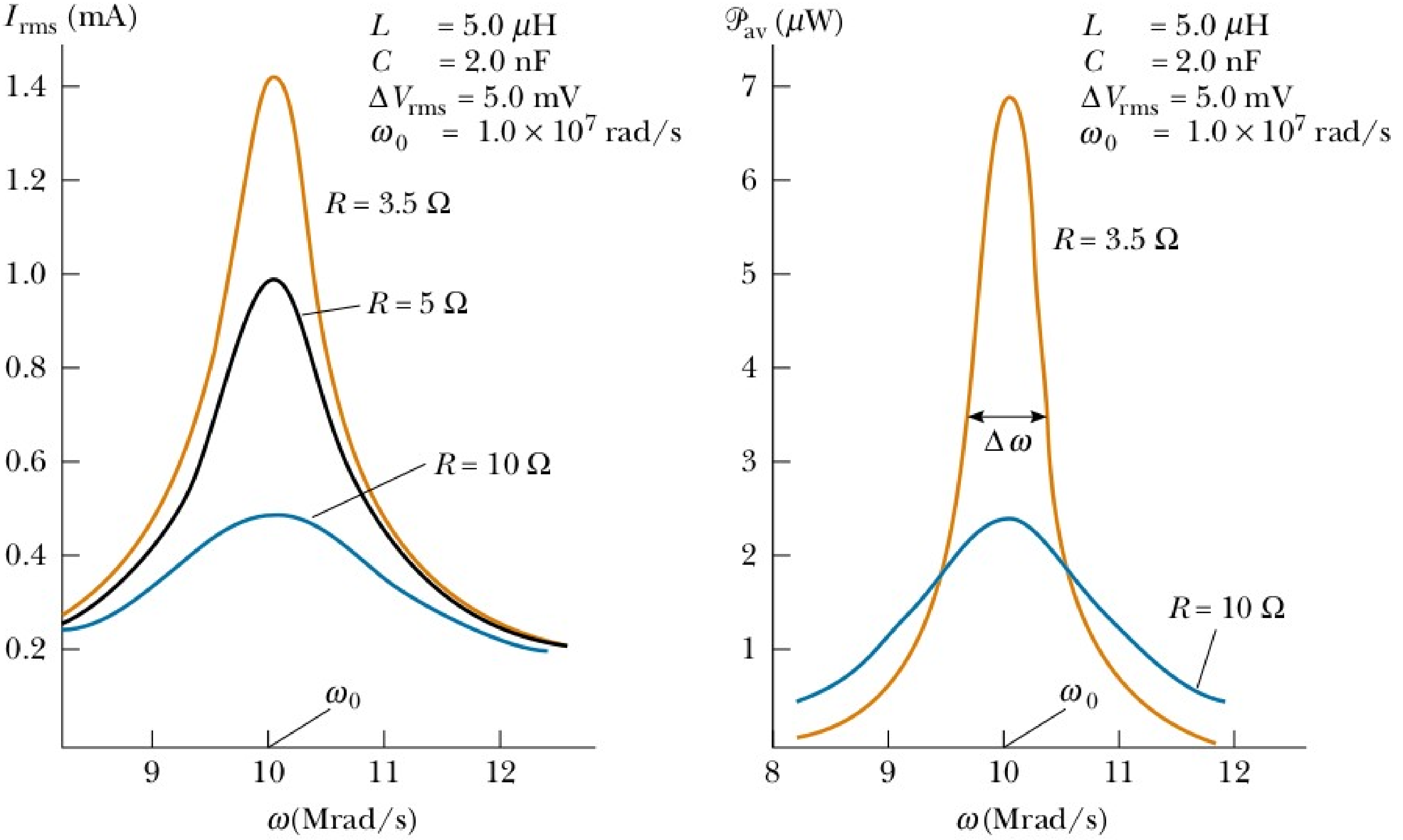
$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Series LCR circuit

Band Width.



Series LCR circuit

- See book Waves and Oscillation by N K Bajaj

- Resonance
- Power Dissipation
- Quality Factor
- Band Width.

Parallel LCR Circuit.

See book :

- Waves and Oscillation by N K Bajaj
- Electricity and Magnetism by Mahajan and Rangwala