Semester II

CC- III: ELECTRICITY AND MAGNETISM

(Credits: Theory-04, Practicals-02)

F.M. = 75 (Theory - 40, Practical – 20, Internal Assessment – 15)

Internal Assessment [Class Attendance (Theory) – 05, Theory (Class Test/ Assignment/ Tutorial) – 05, Practical (Sessional Viva-voce) - 05]

Theory: **60 Lectures**

Electric Field and Electric Potential

Electric field: Electric field lines. Electric flux.Gauss' Law with applications to charge distributions with spherical, cylindrical and planar symmetry. (6 Lectures)

What is electric field?

- A static creates a field about its surroundings. This vector field is called an “Electric field”
- A moving charge particle generates electric field as well as magnetic field

References: Introduction to Electrodynamics, D J Griffiths

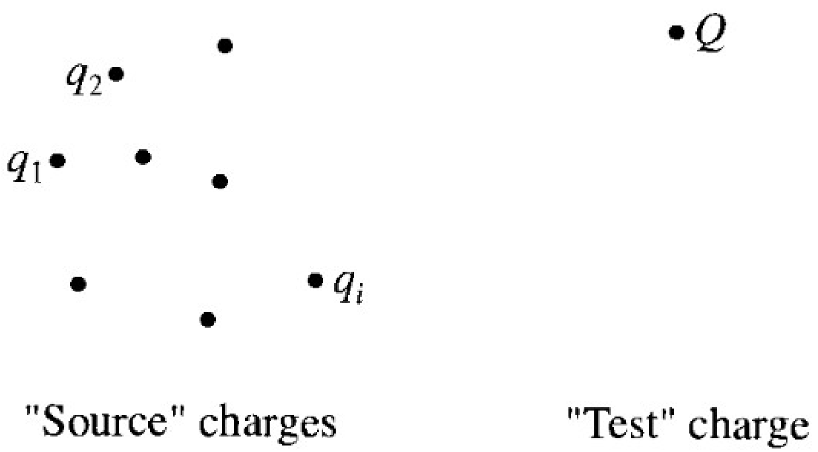
Electrostatic

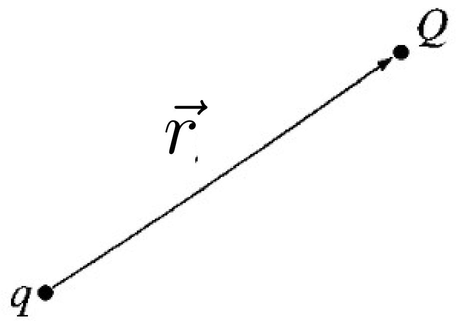
- Here we study static electric charge
- The electric field is also static. i.e. it does not change with time.

Electrostatic

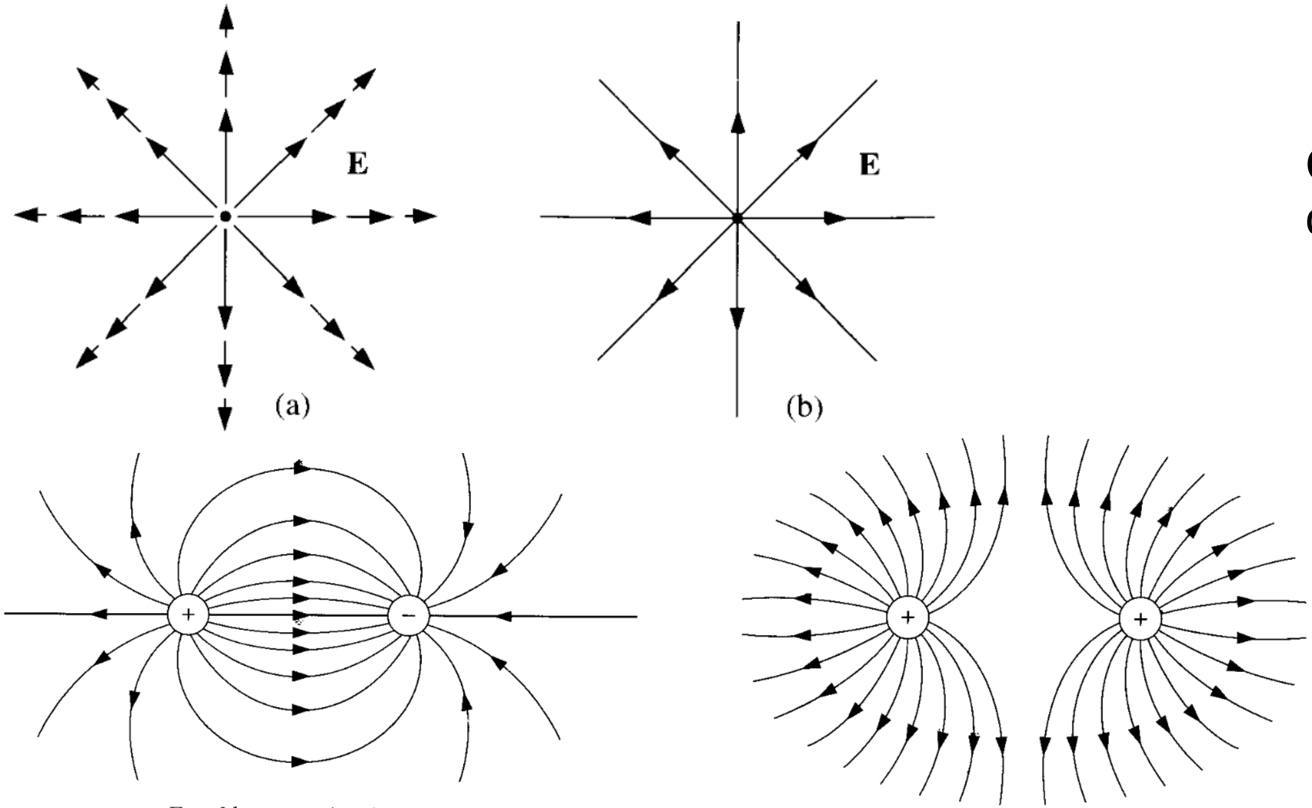
Electric charge which does not move and the electric field is time independent

$$\vec{F} = \frac{Qq}{4\pi\epsilon_0 r^2}\hat{r}$$





Electric filed lines

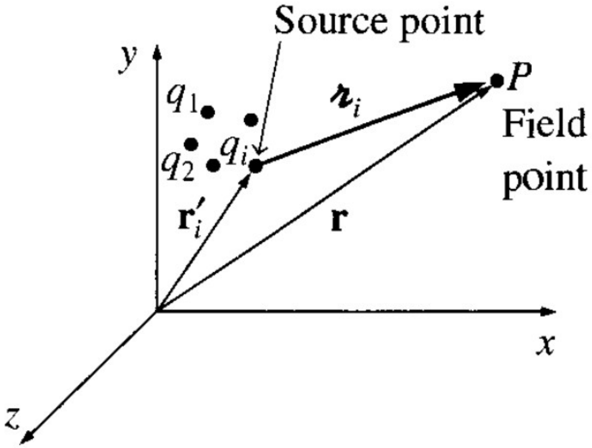


Equal but opposite charges

Equal charges

Can you see divergence of electric field lines?

Force and Field



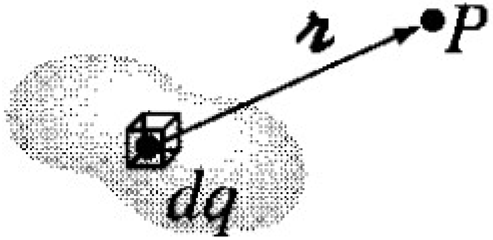
$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{\mathscr{r}_1^2}\hat{\boldsymbol{\mathscr{r}}}_1 + \frac{q_2 Q}{\mathscr{r}_2^2}\hat{\boldsymbol{\mathscr{r}}}_2 + \ldots\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1\hat{\boldsymbol{\mathscr{r}}}_1}{\mathscr{r}_1^2} + \frac{q_2\hat{\boldsymbol{\mathscr{r}}}_2}{\mathscr{r}_2^2} + \frac{q_3\hat{\boldsymbol{\mathscr{r}}}_3}{\mathscr{r}_3^2} + \ldots\right),$$

$$\boxed{\mathbf{F} = Q\mathbf{E},}$$

$$\mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i.$$

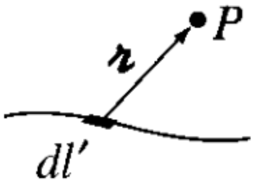
Continuous Charge Distribution



(a) Continuous distribution

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,dq.$$

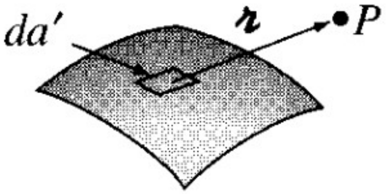
Continuous Charge Distribution



(b) Line charge, λ

Thus the electric field of a line charge is

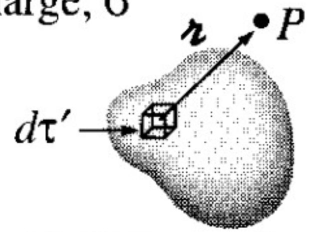
$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}} \frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, dl';$$



(c) Surface charge, σ

for a surface charge,

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{S}} \frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, da';$$



(d) Volume charge, ρ

for a volume charge,

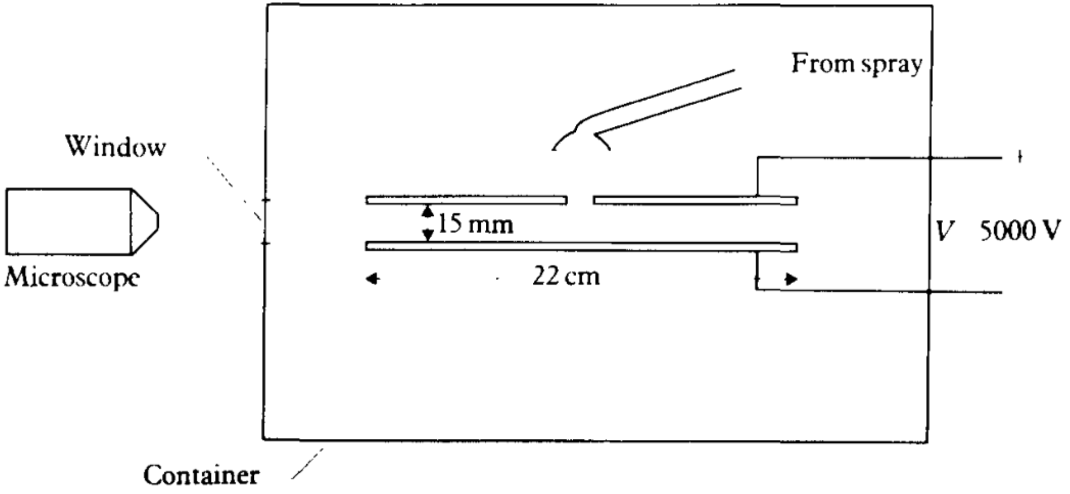
$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, d\tau'.}$$

About ELECTRON

Charge of Electron
C = 1.6×10^{-19} Coulomb

Mass of electron
9.11×10^{-31} Kg

Millikan Oil Drop Experiment (Year 1909)



$$Mg = 6\pi\eta r v_1$$

$$M = \frac{4}{3}\pi r^3(\rho_O - \rho_A)$$

$$q\frac{V}{D} - Mg = 6\pi\eta r v_2$$

$$q = 6\pi\eta r\left(\frac{D}{V}\right)(v_1 + v_2)$$

He got !

$$|q| \simeq 1.59\, n \times 10^{-19}\ \mathrm{C}$$

What have we learn?

Electric field is created by static charged particles..

Electric Force obey superposition principle

$F = F_1 + F_2 + F_3$ ..

Unit of

- Force ~ Newton
- Charge ~ Coulomb
- Electric Field ~ Newton/Coulomb
- Potential ~ Newton/(Coulomb x meters)

Charge distribution creates electric field

Discrete charges

Continuous charge distribution

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,dq.$$

Sample Questions: Find electric field?

Example 2.1

Find the electric field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform line charge λ (Fig. 2.6).

Ans

$$d\mathbf{E} = 2\frac{1}{4\pi\epsilon_0}\left(\frac{\lambda\, dx}{\mathscr{r}^2}\right)\cos\theta\,\hat{\mathbf{z}}.$$

$$E = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{2\lambda z}{(z^2+x^2)^{3/2}}dx$$

$$= \frac{2\lambda z}{4\pi\epsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]\Bigg|_0^L$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}},$$

$$E \cong \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z^2},$$ for $z >> L$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{z};$$ for $L >> z$

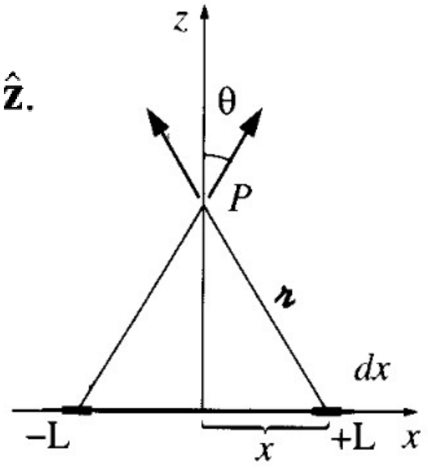


Figure 2.6

Sample Questions: Find electric field?

Problem 2.5 Find the electric field a distance z above the center of a circular loop of radius r (Fig. 2.9), which carries a uniform line charge λ.

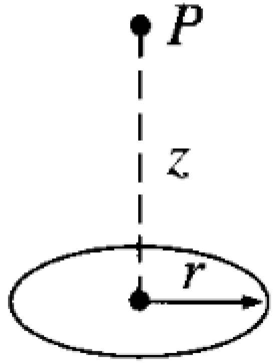


Figure 2.9

Sample Questions: Find electric field?

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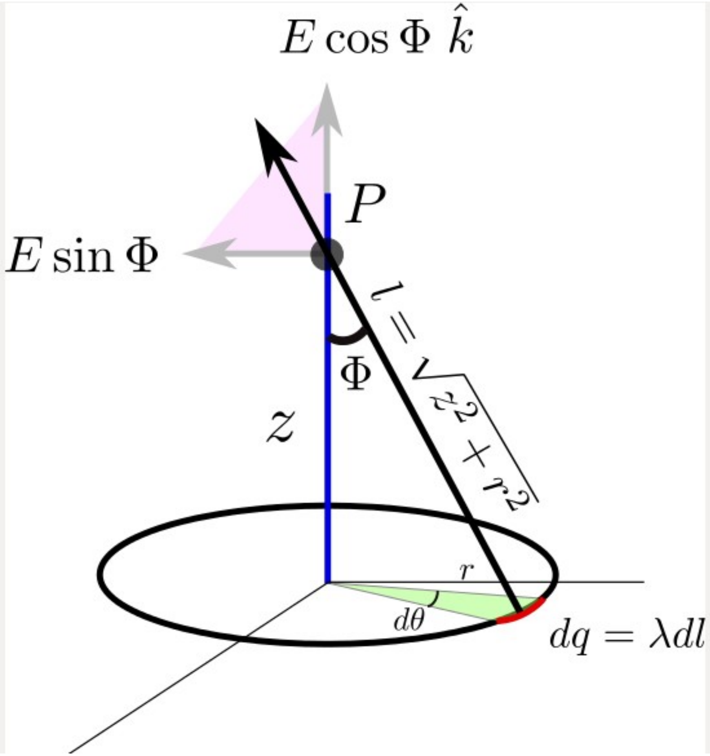

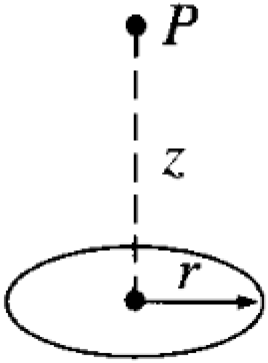

Figure 2.9

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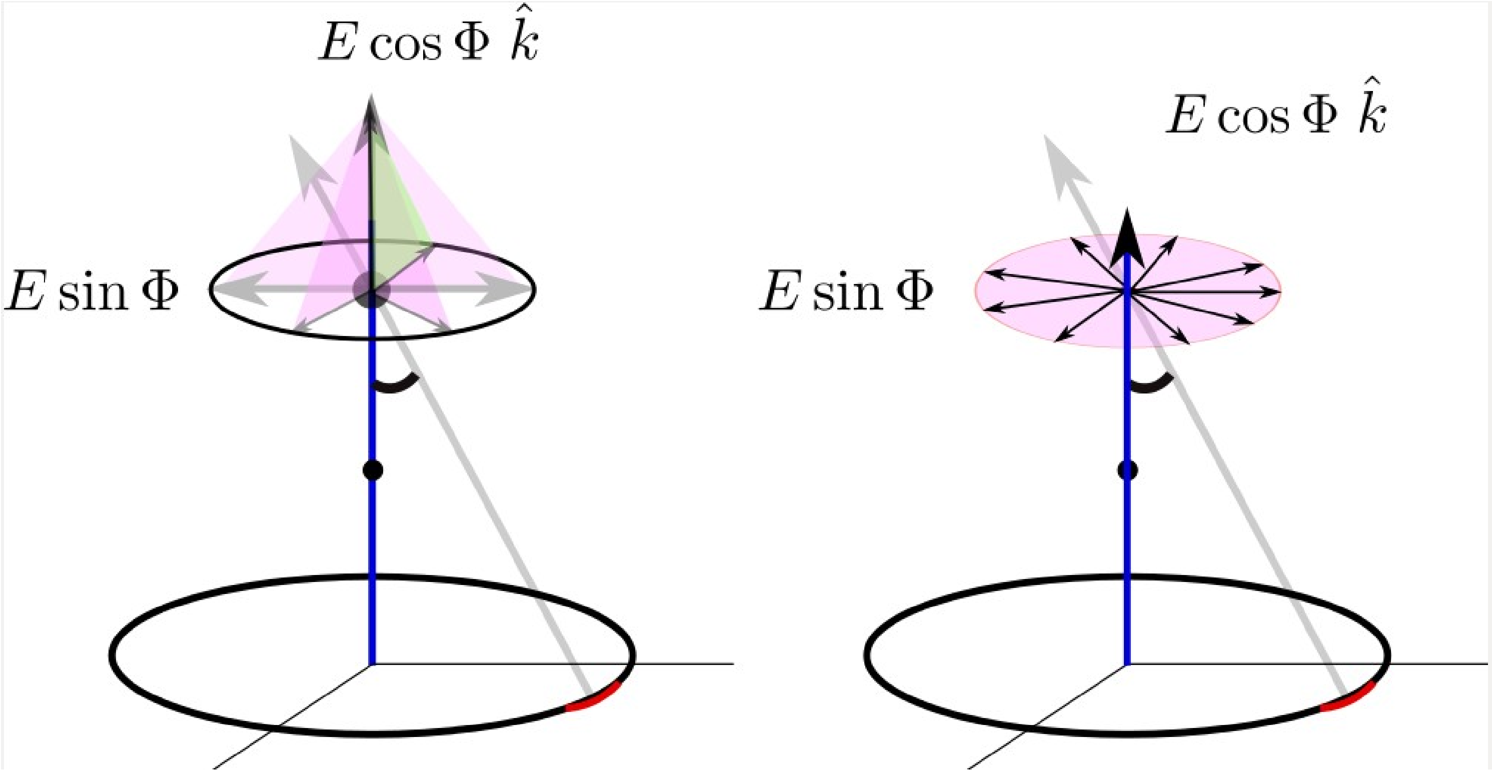


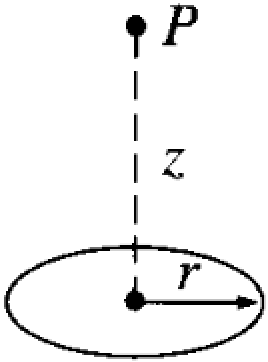


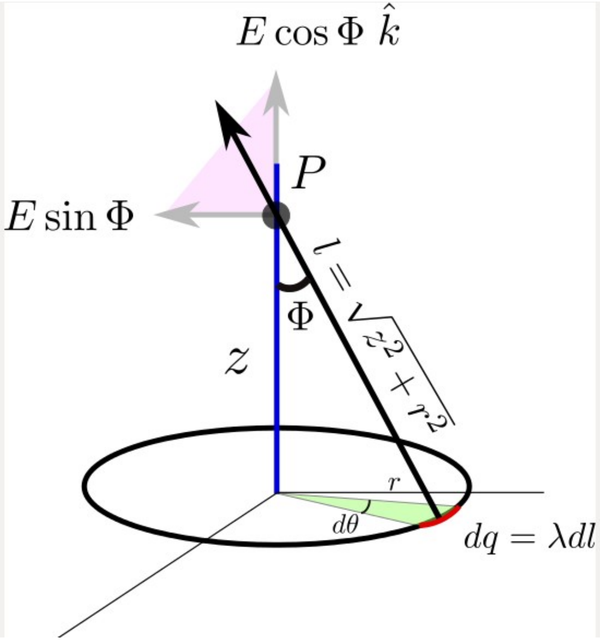
Figure 2.9

Sample Questions: Find electric field?

Problem 2.5 Find the electric field a distance z above the center of a circular loop of radius r (Fig. 2.9), which carries a uniform line charge λ.

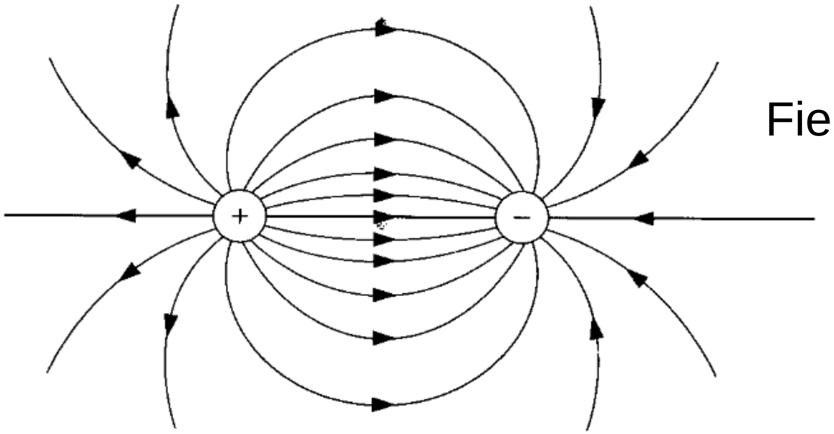
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^{2\pi}\frac{(\cos\Phi)\ \lambda r d\theta}{(z^2+r^2)}\hat{k}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{zr\lambda 2\pi}{(z^2+r^2)^{3/2}}$$

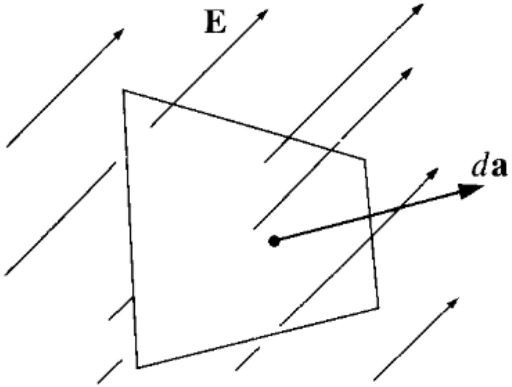


Check yourself

Flux



Field Lines



Flux of electric field through surface ***S***

$$\Phi_E \equiv \int_S \mathbf{E} \cdot d\mathbf{a},$$

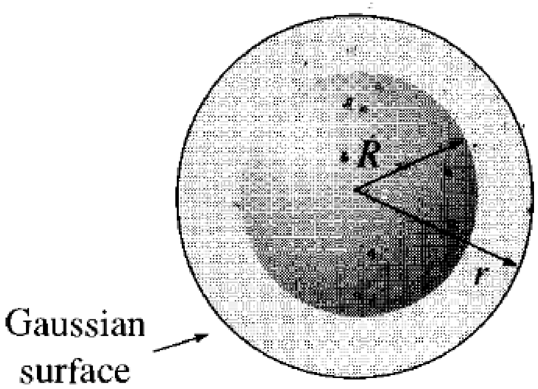
Flux is measure of number of filed lines..

Flux

$$\oint \mathbf{E} \cdot d\mathbf{a} = \int \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{\mathbf{r}}\right) \cdot (r^2 \sin\theta \, d\theta \, d\phi \, \hat{\mathbf{r}}) = \frac{1}{\epsilon_0} q.$$

For any closed surface:

$$\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}},$$



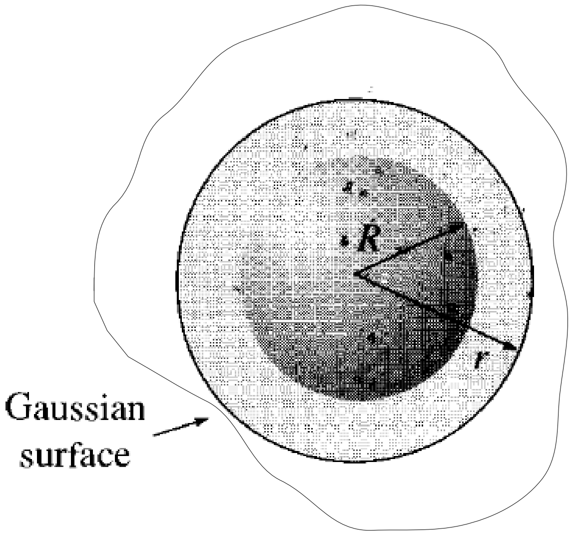
This is Integral form of Gauss's Law

Flux

$$\oint \mathbf{E} \cdot d\mathbf{a} = \int \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r^2} \hat{\mathbf{r}} \right) \cdot (r^2 \sin\theta \, d\theta \, d\phi \, \hat{\mathbf{r}}) = \frac{1}{\epsilon_0} q.$$

For any closed surface:

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}},$$



This is Integral form of Gauss's Law

Important thing is that we do not need a spherical shape to enclose the charged objects !!

Flux: Divergence theorem

For any vector **E**

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau.$$

ρ is Charge density

$$Q_{\text{enc}} = \int_{\mathcal{V}} \rho\, d\tau.$$

$$\int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau = \int_{\mathcal{V}} \left(\frac{\rho}{\epsilon_0}\right) d\tau.$$

This is Integral form of Gauss's Law

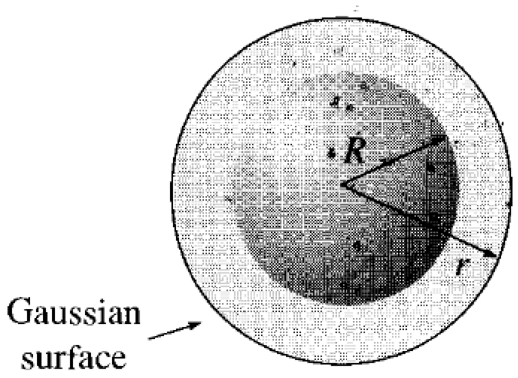
$$\boxed{\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}},}$$

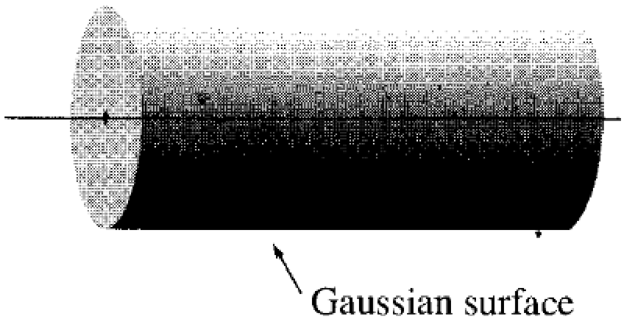
This is Differential form of Gauss's Law

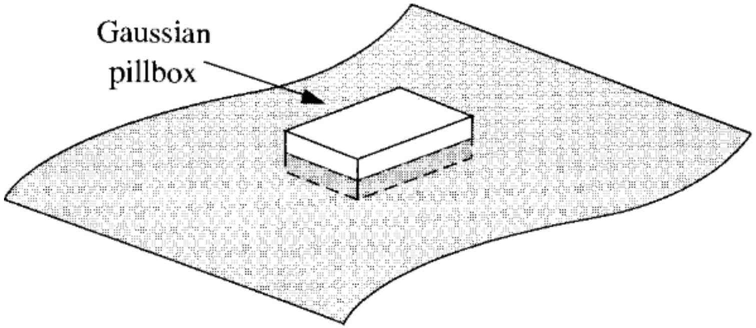
$$\boxed{\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho.}$$

Application of Gauss's Law

1. *Spherical symmetry.* Make your Gaussian surface a concentric sphere.
2. *Cylindrical symmetry.* Make your Gaussian surface a coaxial cylinder (Fig. 2.19).
3. *Plane symmetry.* Use a Gaussian "pillbox," which straddles the surface (Fig. 2.20).

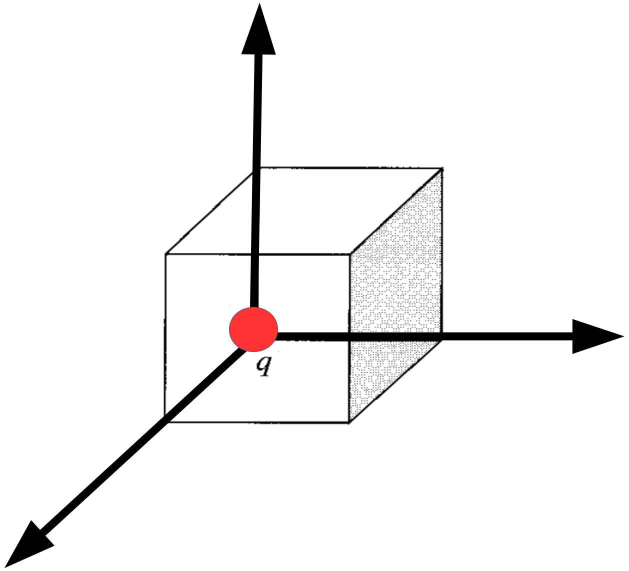






Problems?

- **A charge q sits at the corner of a cube as shown in figure below. What is the flux of E** through the shaded area?



Problems?

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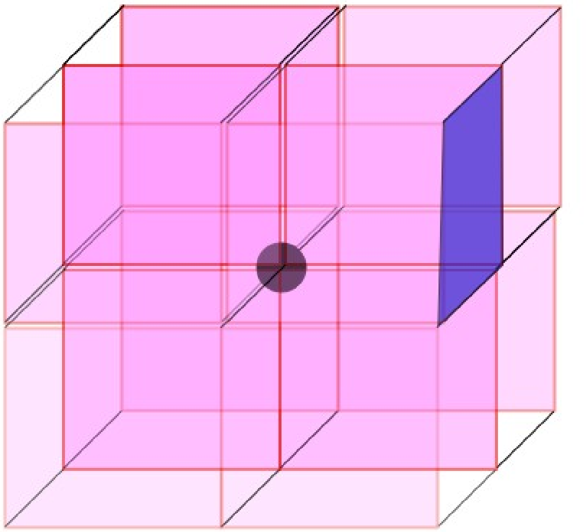
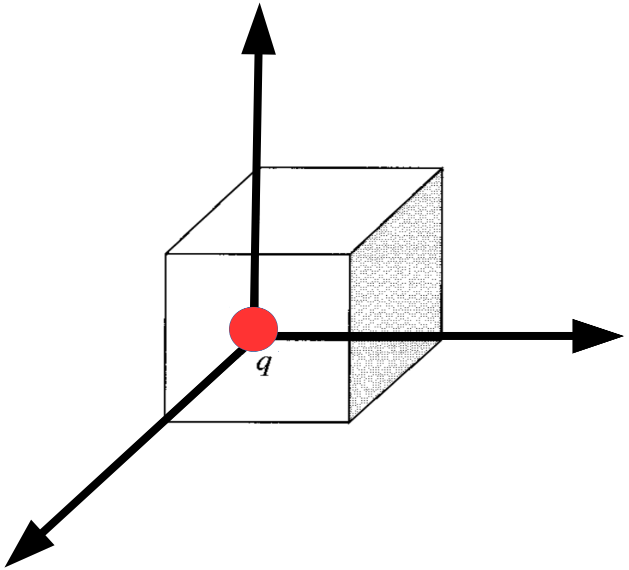


total 8 cubes faces the flux
flux through one cube is

$$\frac{1}{8}\frac{q}{\epsilon_0}$$



flux through the shaded surface = $\frac{1}{24}\frac{q}{\epsilon_0}$

Problems?

Example 2.2

Find the field outside a uniformly charged solid sphere of radius R and total charge q.